Controller Design for Integrating and Runaway Processes Involving Time Delay

This paper presents an efficient method for the design of controllers for integrating and runaway processes. The method is based on model matching in the frequency domain. The presence of open-loop instability as well as pure time delay in the process models make the design task challenging for these classes of processes.

The goal is to achieve low-order, easily implementable cascade controllers in a unity-output-feedback configuration. It is shown that the central problem is in the selection of appropriate reference models. Several key constraints are developed which relate a given process model to a class of reference models for achieving total stability. Typical design examples are presented to clearly illustrate the various mathematical techniques.

Stanley B. Quinn, Jr.
Frederiksen and Shu Laboratories
Arlington Heights, IL 60005

C. K. Sanathanan
University of Illinois
Chicago, IL 60680

Introduction

Most controller design studies assume the availability of detailed plant models. In areas such as aerospace, a large amount of effort is invested in capturing the microscopic details of the system with a high-order model. In contrast, process models, although of high order, resist this elaborate analysis. Because of nonlinearities, precise high-order linear models for processes are usually, at best, misleading. Furthermore, large uncertainties with respect to the quantification of the various phenomena make the parameters of a detailed model highly susceptible to error. Therefore, easily identifiable macroscopic models are usually employed in the design of process controllers. Unlike aerospace designs, high speed of response is not always critical in process control. All of these factors have been the motivation for the use of process models that consist of low-order transfer functions (TF's) with a sizable dead time. This paper presents novel techniques for designing easily implementable controllers for processes that are open-loop-unstable.

Process models are typically expressed as generic TF's falling into one of three categories (McMillan, 1983). These are:

Self-regulating processes:

$$G_P(s) = \frac{P_0 e^{-s\tau}}{1 + Ts} \tag{1a}$$

Integrating processes:

$$G_P(s) = \frac{P_0 e^{-s\tau}}{s(1 + Ts)} \tag{1b}$$

Runaway processes:

$$G_P(s) = \frac{P_0 e^{-s\tau}}{(1 + T_1 s)(1 - T_2 s)} \tag{1c}$$

The parameters of these macroscopic models are typically determined (identified) experimentally (Eykhoff, 1981; Gustavsson et al., 1977; Sundareson et al., 1978; Graupe, 1984). They are dependent on the predominant process characteristics such as volume, density and flow rate.

Noteworthy in the integrating and runaway models are the closed right half plane (RHP) poles which characterize open-loop instability. In the time domain, this indicates that even inputs of small amplitude will lead to unbounded outputs and, hence, saturation. Feedback is imperative for stabilization but the formulation of satisfactory loop compensation is difficult particularly due to the combined effects of the RHP poles and the time delay. Contemporary controller design techniques that are based on the Smith predictor, such as internal model control, are intrinsically inapplicable to unstable processes (Morari and Doyle, 1986). State-space techniques may be used but generally require high-order observers as well as several design iterations to obtain closed-loop stability (Stahl and Hippe, 1987).

Regardless of the order and sophistication of $G_P(s)$, the goal is to provide simple, easily implementable controllers with a transparent design procedure. That is, a procedure which explicitly incorporates the design specifications and evaluates them in terms of the required controller complexity. Such a procedure is available for stable delayed processes, of low or high order (Sanathanan and Quinn, 1987). For the sake of clarity and completeness, a summary of the method is presented here. Subsequently, techniques to deal with unstable, delayed processes are presented.

Controller Design Procedure

The basic procedure is to first select a reference model, $G_R(s)$, which satisfies the design specifications for the closed-loop system. A low-order controller *TF*, $G_C(s)$, is then computed such that the processs, $G_P(s)$, when placed in the unity-feedback configuration of Figure 1, will closely match the reference model. The controller TF which yields an exact match is given by the Guillemin-Truxal synthesis equation (D'Azzo and Houpis, 1981; Peczkowski and Sain, 1977).

$$K(s) = \frac{G_P(s)^{-1}}{G_R(s)^{-1} - 1} \tag{2}$$

Unfortunately, the controller of Eq. 2 is unsuitable for practical implementation due to the inverted time delays. Even if the delays are approximated rationally and the overall *TF* reduces to a causal form, the controller order will be unrealistically high. The solution is to synthesize $G_C(j\omega)$, a low-order rational approximation of $K(j\omega)$, in the frequency domain. The exact frequency responses of the process and reference time delays, $e^{-j\omega\tau}$, are used in Eq. 2. The complete procedure and several illustrative examples are given in Sanathanan and Quinn (1987).

An important attribute of the design procedure, as outlined above, is that the poles and zeros of the plant are not explicitly cancelled by the controller, as implied in Eq. 2. Rather, the frequency domain behavior of $K(s)$ is captured in the important low frequency range in which $G_R(j\omega)$ is characterized. While some poles are approximately cancelled, others can be forced to migrate to more desirable s-plane locations by an appropriate choice of the reference model, $G_R(s)$. The choice of such reference models is the central theme of this paper.

Model matching design techniques essentially replace the dynamics of the plant or process with those of the reference model. Because the high-frequency response of the controller must be finite, the reference model must have a denominator polynomial degree which is greater than its numerator degree by at least as much as the corresponding process degree difference. That is, the *pole-zero* excess of the reference model must be at least as great as that of the process model.

Theoretically, one might cancel unwanted poles and zeros by prefiltering the plant input with their inverse. This presents two practical difficulties.

• 1. Exact cancellation is virtually impossible due to model parameter uncertainties. Significant time domain mismatch can be thus introduced in attempts to cancel highly underdamped poles.

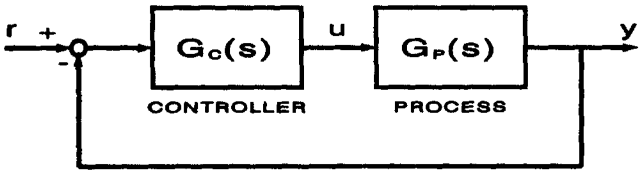

Figure 1. Unity-feedback configuration for process control.

• 2. The cancellation of RHP poles and zeros precludes *total stability*.
Even if the cancellation is exact, *unstable hidden modes* remain which can be excited by initial conditions or exogenous inputs (Chen, 1987). It is for this reason that design techniques which rely on the cancellation of dominant plant dynamics cannot handle unstable processes.

In order to avoid these problems with the controller design, several guidelines have been adopted for reference model selection.

• 1. Any *RHP* zeros in the rational part of $G_P(s)$ must also appear in $G_R(s)$. Otherwise, the application of Eq. 2 would introduce corresponding unstable poles in $K(s)$.

• 2. The time delay of $G_R(s)$ must be equal to the effective delay in $G_P(s)$. To shorten it implies a noncausal controller and to lengthen it requires the addition of unnecessary extra delay.

• 3. $G_R(s)$ must be specified in such a way that any lightly damped or unstable poles in $G_R(s)$ migrate to the *LHP* when the loop is closed. A flexible approach to simultaneously satisfy these requirements is developed for integrating and runaway processes in the sections that follow.

Reference Models for Integrating Processes

It is well known that control loops which incorporate processes with pure integration and zero dead time can be readily designed to achieve zero steady-state error. The closed-loop output will track step changes in the set point which implies that the overall system *TF* is unity at $\omega = 0$. Consequently, a reference model for such processes must have unity gain or else the controller will need to directly cancel the process integrator. Such cancellation is generally impractical due to system offsets and disturbances. An unstable hidden mode would arise in the form of an offset at $\omega = 0$. The appropriate engineering solution is to specify the reference model such that the poles at $s = 0$ due to each integrator migrate to desirable *LHP* locations when the loop is closed.

This imposes some interesting subtleties on the closed-loop system and, hence, the reference model. The type of a loop, ν, is equal to the order of integration; the number of open-loop poles at $s = 0$. For an undelayed loop, the transmission (return ratio) is given by a ratio of polynomials:

$$L(s) = G_P(s)G_C(s) = \frac{(\alpha_0 + \alpha_1 s + \cdots)}{s^\nu(\gamma_0 + \gamma_1 s + \cdots)} \tag{3}$$

When this loop is closed with unity feedback, the resulting transfer function is given by $G_{CL}(s)$.

$$G_{CL}(s) = \frac{L(s)}{1 + L(s)} = \frac{\alpha_0 + \alpha_1 s + \alpha_2 s^2 + \cdots}{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots}, \tag{4a}$$

where

$$\alpha_\iota = \beta_\iota \ \forall \ \iota < \nu \tag{4b}$$

Therefore, a reference model for a type-ν loop must possess the coefficient relationship of Eq. 4b. However, this relationship does not, in general, hold for loops that contain dead time.

If a type-one loop is required, the reference model implication is unity steady-state gain. Because the delay vanishes in the steady-state, the closed-loop model, as in the undelayed case, must simply have $\alpha_0 = \beta_0$. If a loop type of greater than one is required, for tracking ramp inputs or rejecting disturbances to an integrating process, the situation becomes more complicated.

Consider the delayed reference model $G_R(s)$.

$$G_R(s) = \frac{(\alpha_0 + \alpha_1 s + \cdots)e^{-s\tau}}{\beta_0 + \beta_1 s + \beta_2 s^2 + \cdots} \tag{5}$$

This will be matched by a unity feedback control system with a type-ν loop transmission,

$$L(s) = \frac{(\alpha_0 + \alpha_1 s + \cdots)e^{-s\tau}}{s^\nu(\gamma_0 + \gamma_1 s + \cdots)} \tag{6}$$

if and only if:

$$\beta_0 + \beta_1 s + \cdots + \beta_{\nu-1}s^{\nu-1} = (\alpha_0 + \alpha_1 s + \cdots + \alpha_{\nu-1}s^{\nu-1})e^{-s\tau} \tag{7}$$

In order to examine the conditions under which Eq. 7 is satisfied, write $e^{-s\tau}$ as

$$e^{-s\tau} = \frac{1 - \delta_1 s + \delta_2 s^2 - \cdots}{1 + \delta_1 s + \delta_2 s^2 + \cdots} = \frac{e^{-s\tau/2}}{e^{+s\tau/2}} \tag{8}$$

Note that this is not to be taken as an approximation but an exact representation as the ratio of two infinite series. The coefficients, δ_ι, can be supplied by the familiar MacLaurin series.

$$e^{s\tau/2} = 1 + s\tau/2 + s^2\tau^2/8 + s^3\tau^3/48 + \cdots \Longrightarrow \delta_\iota = (\tau/2)^\iota/\iota! \tag{9}$$

Therefore, in order to satisfy Eq. 7, $G_R(s)$ must be selected such that

$$\sum_{l=0}^{\iota} \beta_l \delta_{\iota-l} = \sum_{l=0}^{\iota} \alpha_l \delta_{\iota-l}(-1)^{(\iota-l)}, \forall \iota \leq \nu - 1 \tag{10}$$

The implications for a type-two loop are that $\alpha_0 = \beta_0$ and $\alpha_1 = \beta_1 + \beta_0\tau$. In general,

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{\nu-1} \end{bmatrix} = \begin{bmatrix} \delta_0 & 0 & \cdot & \cdot & 0 \\ -\delta_1 & \delta_0 & 0 & \cdot & 0 \\ \delta_2 & -\delta_1 & \delta_0 & 0 & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & 0 \\ (-1)^{\nu-1}\delta_{\nu-1} & \cdot & \cdot & \cdot & \delta_0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \delta_0 & 0 & \cdot & \cdot & 0 \\ \delta_1 & \delta_0 & 0 & \cdot & 0 \\ \delta_2 & \delta_1 & \delta_0 & 0 & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & 0 \\ \delta_{\nu-1} & \cdot & \cdot & \cdot & \delta_0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{\nu-1} \end{bmatrix} \tag{11}$$

Thus, once the denominator polynomial has been selected, the numerator will be constrained, according to Eq. 11, by the loop type.

This mechanical exercise in coefficient selection has the potential to significantly affect the time response. Although α_0 equals β_0, α_1 is greater than β_1. Therefore, the most dominant *TF* zero tends to be at a lower frequency than the dominant pole(s). This introduces overshoot into the step response of the closed-loop system. This phenomenon must be recognized as a characteristic of delayed loops in general, not as a byproduct of this design procedure. The above analysis holds true whether $G_R(s)$ of Eq. 5 is a reference model for design or the actual closed-loop *TF* for an arbitrary delayed $L(s)$.

In order to avoid overshoot in such a situation, the burden must fall on the set point strategy. If small gradual changes are all that will be made to the input, the dominant low-frequency zero will not present a problem. If engineering considerations require the control hardware to insure this, the set point change can be passed through a simple filter with a single pole at the frequency of the problematic zero.

The controller design procedure for integrating processes is summarized in the following steps.

1. Select the reference model denominator polynomial; the poles of $G_R(s)$.
2. Determine values of the ν low-order numerator coefficients in terms of the selected denominator polynomial and Eq. 11.
3. Specify any remaining numerator coefficients, thus completing the *TF* for $G_R(s)$ (the time delay equals that of the process model).
4. Substitute the frequency responses of $G_P(s)$ and $G_R(s)$ into Eq. 2 to determine the ideal controller response, $K(j\omega)$.
5. Synthesize $G_C(s)$, a low-order *TF* which approximates the ideal controller response.

Example 1. Consider the following model for an integrating process. Its regulation might represent, for example, a liquid level control.

$$G_P(s) = \frac{0.5e^{-\tau s}}{s(1 + Ts)}, \quad T = 5, \tau = 2$$

The system is known to be subject to steady-state disturbances at the plant input. The output, however, must exhibit zero steady-state error with respect to the reference set point. An additional integrator is therefore needed in the controller and the overall loop will be of type two.

The reference model must have a pole-zero excess of at least two and a numerator degree of at least one. The minimal denominator degree is thus three. A reference model that satisfies these constraints, as well as Eq. 11 for $\nu = 2$, is given by $G_R(s)$.

$$G_R(s) = \frac{1 + 32s}{(1 + 10s)^3}e^{-2s}$$

The step response exhibits about 30% overshoot. The corresponding rational controller, synthesized to match the frequency response of Eq. 2, is given by $G_C(s)$.

$$G_C(s) = \frac{0.005525(1 + 31.75s)(1 + 5.342s)}{s(1 + 2.811s)}$$

The step response of the closed-loop system using this controller is shown in Figure 2 along with the step response of $G_R(s)$.

The closed-loop system matches the reference model quite well. If the overshoot is unacceptable, a simple first-order filter can be appended. The overall configuration is shown in Figure 3. The filter is not in the loop and therefore has no effect on $L(s)$ and the constraints of Eq. 7. The filter frequency, ω_F, is selected to cancel the dominant *LHP* zero of $G_R(s)$. Virtually exact cancellation is possible because this frequency, introduced by the controller, is known precisely. In this case, $\omega_F = 0.03150 \approx 1/32$. The filtered reference response is now given by $G_F(s)$.

$$G_F(s) \approx \frac{e^{-2s}}{(1+10s)^3} = \frac{1}{1+s/\omega_F} G_R(s)$$

The step responses of $G_F(s)$ and the overall system, as shown in Figure 4, match virtually exactly.

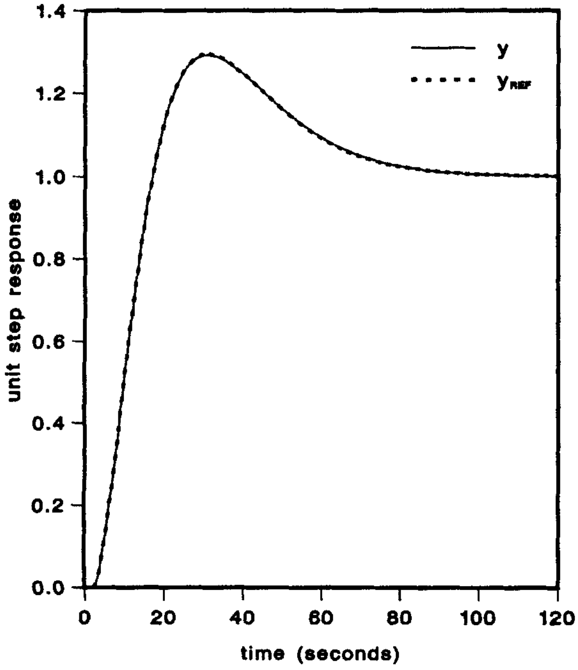

Figure 2. Step responses for example 1.

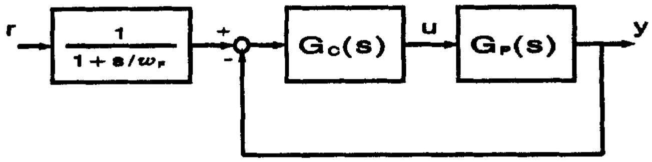

Figure 3. Filtered set point control configuration.

Reference Models for Unstable Processes without Delay

A mathematical technique has been developed in order to determine reference responses for unstable rational processes (no pure time delay) that insure total stability. The basic procedure for arriving at such references is given below. For a more comprehensive development, including proofs of the essential theoretical assertions, see Quinn 1988, and Quinn and Sanathanan (1988).

The plant *TF*, a ratio of polynomials, is initially factored according to open *LHP* (−) and closed *RHP* (+) numerator and denominator roots.

$$G_P(s) = \frac{N(s)}{D(s)} = \frac{N_-(s)N_+(s)}{D_-(s)D_+(s)},$$
$$\deg[N_-(s)] = j, \deg[D_-(s)] = k, \deg[N_+(s)] = m, \deg[D_+(s)] = n \quad (12)$$

To insure closed-loop stability robustness, underdamped *LHP* plant poles that are close to the $j\omega$ axis are included in $D_+(s)$, the poles that must migrate to the left. The reference model, as yet undetermined, is also a ratio of polynomials.

$$G_R(s) = A_N(s)/A_D(s) \quad (13)$$

Now, the first design step is to completely specify the reference denominator polynomial (stable closed-loop poles) with the following degree constraint:

$$i = \deg[A_D(s)] \geq 2n + k - j - 1 \quad (14)$$

Recall that the reference numerator must include any *RHP* zeros of $G_P(s)$.

$$A_N(s) = F(s)N_+(s)$$
$$\text{polynomial } F(s) \text{ to be determined} \quad (15)$$

In order to select $F(s)$ for total stability the *Diophantine equation* must be solved for $F(s)$ and another polynomial, $G(s)$.

$$F(s)N_+(s) + G(s)D_+(s) = A_D(s) \quad (16)$$

Any totally stable unity-feedback system which incorporates

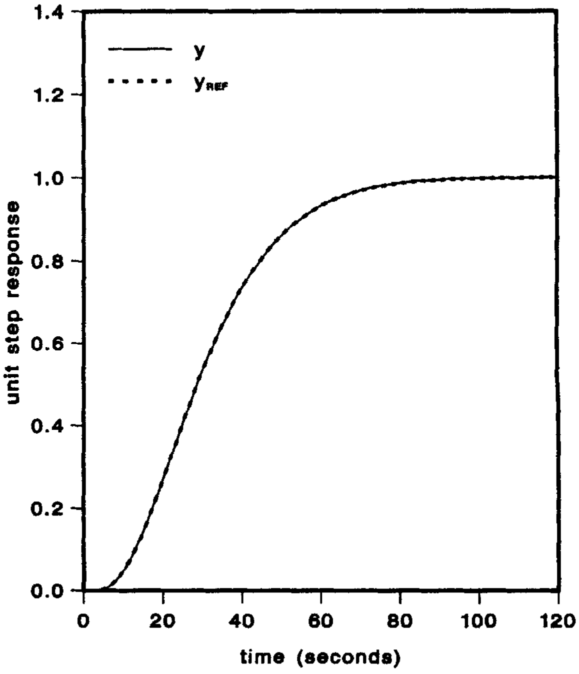

Figure 4. Step responses for filtered system, example 1.

$G_P(s)$ *must* satisfy this relationship. Likewise, satisfaction of the Diophantine Eq. 7 is sufficient to guarantee migration of the $D_+(s)$ poles into the *LHP*.

Let

$$A_D(s) = a_0 + a_1 s + \cdots + a_i s^i \tag{17a}$$

$$N_+(s) = b_0 + b_1 s + \cdots + b_m s^m \tag{17b}$$

$$D_+(s) = d_0 + d_1 s + \cdots + d_n s^n \tag{17c}$$

$$F(s) = f_0 + f_1 s + \cdots + f_{n-1} s^{n-1} \tag{17d}$$

$$G(s) = g_0 + g_1 s + \cdots + g_{i-n} s^{i-n} \tag{17e}$$

Equation 16 can then be expressed as a system of linear algebraic equations.

$$\begin{bmatrix} b_0 & 0 & & 0 & d_0 & 0 & & 0 \\ b_1 & b_0 & 0 & \cdot & d_1 & d_0 & 0 & \cdot \\ \cdot & b_1 & \cdot & \cdot & d_2 & d_1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot \\ b_m & \cdot & \cdot & b_0 & \cdot & \cdot & \cdot & 0 \\ 0 & b_m & \cdot & b_1 & d_n & \cdot & \cdot & d_0 \\ \cdot & 0 & \cdot & \cdot & 0 & d_n & \cdot & d_1 \\ \cdot & & & \cdot & & 0 & \cdot & \cdot \\ \cdot & & & b_m & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & & \cdot \\ 0 & \cdot & \cdot & 0 & 0 & & 0 & d_n \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \cdot \\ \cdot \\ g_0 \\ g_1 \\ \cdot \\ \cdot \\ \cdot \\ g_{i-n} \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ a_i \end{bmatrix} \tag{18}$$

A unique minimum-degree solution, $\{F_0(s), G_0(s)\}$, is available to Eq. 16 when the following degree assignments are made (Quinn, 1988; Kucera, 1979):

$$\deg\,[F_0(s)] = n - 1 \tag{19a}$$

$$\deg\,[G_0(s)] = i - n \tag{19b}$$

A reference model can now be completely determined in terms of the selected poles, the *RHP* plant zeros, and the Diophantine solution, $\{F_0(s), G_0(s)\}$.

$$G_R(s) = \frac{F_0(s)N_+(s)}{A_D(s)} \tag{20}$$

Some flexibility may be desirable in the determination of $A_N(s)$, however. Steady-state closed-loop gain or error are examples of characteristics which might be specified in terms of the low-order coefficients of $A_N(s)$ in conjunction with $A_D(s)$. Several numerator coefficients can be so specified by manipulating $F(s)$ according to the general solution of Eq. 16.

$$F(s) = F_0(s) + E(s)D_+(s) \tag{21a}$$

$$G(s) = G_0(s) - E(s)N_+(s) \tag{21b}$$

That is, for each coefficient to be fixed in $A_N(s)$, the degree of $F(s)$ is increased and the new polynomial, $E(s)$, is calculated to bring about these coefficients according to Eqs. 15 and 21a. $E(s)$ can then be freely substituted into Eq. 21 to yield $\{F(s), G(s)\}$ which satisfies Eq. 16, as long as:

$$\deg\,[E(s)] \le i + j - k - 2n \tag{22}$$

Note that $i = \deg\,[A_D(s)]$ may need to be increased in order to allow this.

With the reference model $G_R(s) = F(s)N_+(s)/A_D(s)$ so determined, Eq. 2 again represents the ideal controller:

$$K(s) = \frac{F(s)D_-(s)}{G(s)N_-(s)} \tag{23}$$

As before, its frequency response is computed and matched with a low-order controller *TF* to be implemented in the closed-loop system. Total stability is achieved because a reference model which satisfies Eq. 16 causes all unstable plant poles to migrate to the *LHP* (roots of $A_D(s)$] when the loop is closed. The technique is extended to include plant and reference delays (irrational *TF*'s) in the next section.

Reference Models for Unstable Delayed Processes

An important problem in the process industry is the control of unstable or runaway processes involving pure time delay. In order to apply the controller synthesis techniques of this paper, one must insure that the reference model is specified in such a way that it accounts for the delay in the loop *and* causes the *RHP* process poles to migrate to the LHP rather than be cancelled by compensation zeros. The former is simply a matter of specifying the reference delay to be that of the process. The latter, however, imposes the restrictions of Eq. 16 which are not directly applicable to irrational *TF*'s.

The frequency response of a pure time delay by itself is simply the clockwise transversal of the unit circle in the Nyquist plane. This is equivalent to the frequency response of equal numbers of *LHP* poles and *RHP* zeros which are their mirror images across the imaginary axis. The low frequency behavior of dead time can be approximated with a finite number of such pole-zero pairs, however a relatively large number are required to achieve accuracy sufficient for controller design (Stahl and Hippe, 1987). The strategy of this paper is to use a simple rational approximation for the delay to determine a reference model, $G_R(s)$, which can be matched with total stability. Subsequently, the precise response of the synthesis equation (Eq. 2), with $e^{-j\omega\tau}$ representing dead time, is used to determine the appropriate controller.

The use of a rational approximation makes all of the techniques discussed above applicable to reference model selection. The controller will not attempt to introduce *RHP* poles in order to cancel the nonminimum phase behavior of the delay. As long as the approximation is reasonably accurate in the frequency range of the *RHP* pole(s), their migration to the *LHP*, rather than cancellation, will be brought about. Once an appropriate reference model is selected, the frequency domain synthesis technique is employed using the true delay to determine a practical controller.

The following steps summarize the controller design procedure for runaway processes.

1) Approximate $G_P(s)$ with a rational *TF*. The following second-order *TF* is usually satisfactory for the delay since it is generally shorter than any process time constants (McMillan, 1983).

$$e^{-s\tau} \approx \frac{12 - 6\tau s + \tau^2 s^2}{12 + 6\tau s + \tau^2 s^2} \tag{24}$$

2) Based on the process approximation, select $A_D(s)$ (the reference model denominator polynomial) within the degree constraint of Eq. 14. The poles of the delay approximation are included explicitly.

3) Determine the minimum-degree solution to Eq. 16, $\{F_0(s), G_0(s)\}$ using Eqs. 19 and 18.

4) Compute $E(s)$ such that any desired coefficients of the reference model numerator polynomial, $A_N(s)$ are fixed according to Eqs. 15 and 21a. If this violates Eq. 22, $i = \deg[A_D(s)]$ must be increased and steps 2–4 repeated.

5) Use $E(s)$ to compute the Diophantine solution $\{F(s), G(s)\}$ according to Eq. 21.

6) The reference model is now given by Eq. 20 with the true delay in place of the rational approximation. $G_R(s)$ and $G_P(s)$ are inserted in Eq. 2 to supply the exact-matching controller, $K(s)$.

7) The frequency response of $K(s)$ is used to synthesize the low-order rational controller, $G_C(s)$.

Example 2. Consider an exothermic process, $G_P(s)$, with a time delay of 18 seconds (McMillan, 1983).

$$G_P(s) = \frac{e^{-18s}}{(180s + 1)(1{,}800s - 1)}$$

The following simple approximation of $G_P(s)$ is used to determine $G_R(s)$ by the Diophantine method.

$$G_{\text{APPROX}}(s) = \frac{(324s^2 - 108s + 12)}{(180s + 1)(1{,}800s - 1)(324s^2 + 108s + 12)}$$

$$j = 0, \quad k = 3, \quad m = 2, \quad n = 1$$

A minimal solution requires that $G_R(s)$ be of order four and that $F(s)$ is a constant. Note that since the *RHP* zeros of $G_{\text{APPROX}}(s)$ will be retained in a rational $G_R(s)$, the corresponding poles of the dead-time approximation are specified as roots of the reference denominator. Since the dead time is represented by two poles, the actual reference order is $4 - 2 = 2$. Equal real poles at $\omega = 1/180$ are chosen to give a critically damped response with roughly the speed of the open-loop process.

$$A_D(s) = (1 + 180s)^2(12 + 108s + 324s^2)$$

The minimum-degree solution to Eq. 16 is given by $F_0(s)$ and $G_0(s)$.

$$F_0(s) = 1.222$$

$$G_0(s) = 2.666 + 238.7s + 2{,}012s^2 + 5{,}832s^3$$

The complete reference $G_R(s)$ is now formed using the true delay (G_0 is discarded).

$$G_R(s) = \frac{1.222e^{-18s}}{(1 + 180s)^2}$$

$G_R(j\omega)$ and $G_P(j\omega)$ are now substituted into the synthesis equation and a low-order compensator is synthesized.

$$G_{C1}(s) = \frac{5.500 + 991.1s}{1.0 + 80.65s}$$

The first-order controller, $G_{C1}(s)$, brings about an excellent match as shown by the step responses of the closed-loop system and of $G_R(s)$ in Figure 5.

A possible drawback of this system is that its overall steady-state gain is not unity. This could be adjusted by a simple gain of 1/1.222 in place of the set point filter in the configuration of Figure 3. Note that the overall gain would still be susceptible to disturbances and plant variations.

Fixing the coefficient of s^0 in $A_N(s)$ to equal that of $A_D(s)$ will solve the problem by introducing integration in the loop as discussed earlier in this paper. This can be done by using an $E(s)$ of zero degree in Eq. 21. According to Eq. 22, however, $i = \deg[A_D(s)]$ will have to be increased to five.

The new (approximate) reference denominator is selected as:

$$A_D(s) = (1 + 180s)^3(12 + 108s + 324s^2)$$

Fixing the zero-degree coefficient of $F(s)$ at unity and solving Eqs. 15 and 21 gives:

$$F(s) = 1 + 619.9s$$

$$G(s) = s(742.5 + 44{,}902s + 368{,}000s^2 + 1{,}049{,}800s^3)$$

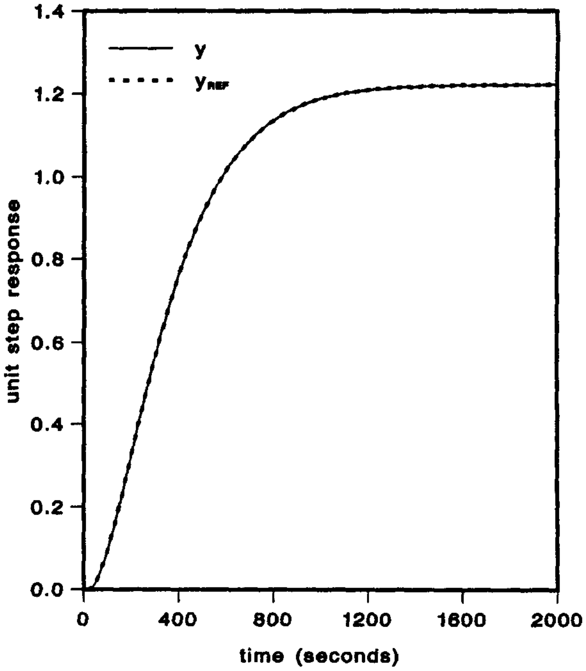

Figure 5. Step responses for example 2: $G_{C1}(s)$ design.

The actual reference *TF* used for controller synthesis is now determined by Eqs. 13 and 15, and the true delay.

$$G_R(s) = \frac{(1 + 619.9s)e^{-18s}}{(1 + 180s)^3}$$

The controller synthesized to give this closed-loop response in conjunction with $G_P(s)$ is:

$$G_{C2}(s) = \frac{0.01616 + 12.93s + 1{,}806s^2}{s(1 + 51.59s)}$$

The reference and closed-loop step responses are shown in Figure 6. Note that the overshoot can easily be eliminated as discussed above.

A conventional PID controller will also suffice. Optimizing the controller parameters to best match the ideal controller frequency response with a PID structure gives $G_{C3}(s)$.

$$G_{C3}(s) = \frac{994s^2 + 14s + 0.015}{s}$$

This also works well, as shown by the step response in Figure 6.

Reference Model Speed of Response

An important design step in the above examples was the selection of the reference model denominator. The dominant roots of this polynomial, the lowest frequency closed-loop poles, dictate the response speed of the closed-loop system. There are several engineering considerations which tend to influence the choice of reference model speed for integrating and runaway processes.

The reference model denominator poles used in example two were simply chosen to be equal to the stable pole frequency of the process model. This worked out well because, as is typical of runaway processes (McMillan, 1983), the unstable pole frequency was much smaller in magnitude. Plant models that have dead time and unstable pole time constants that are roughly the same value as those of the dominant stable poles are potentially problematic in that eligible reference models tend to have high steady-state gains and, hence, low loop gains. This is detrimental in the area of closed-loop robustness. Moreover, these high-gain reference models tend to require controllers that introduce their own underdamped poles into the loop. Because of parameter uncertainty, model matching is usually not exact and these poles thus lead to ringing in the output signal transient response.

The reference model poles for integrating processes can potentially magnify the overshoot effect that results from the low frequency zero that accompanies loops of type greater than one. Referring to Eqs. 5, 10 and 11, if $\alpha_1 = \beta_1 + \beta_0\tau$, then as the dominant time constant (β_1/β_0) becomes small, α_1 approaches $\beta_0\tau$. Thus, while the dominant pole frequency is still increasing, the corresponding zero frequency approaches $1/\tau$. In the time domain, this corresponds to increased overshoot in the step response.

A good rule of thumb is to keep the step response of the reference model at roughly the same speed as the open-loop process. That is, the dominant poles of both should be at about the same frequency. When the reference model poles approach the reciprocal of the dead time, the phenomenon discussed above of underdamped controller poles becomes significant. This overall issue of design considerations related to reference model response speed is currently under investigation.

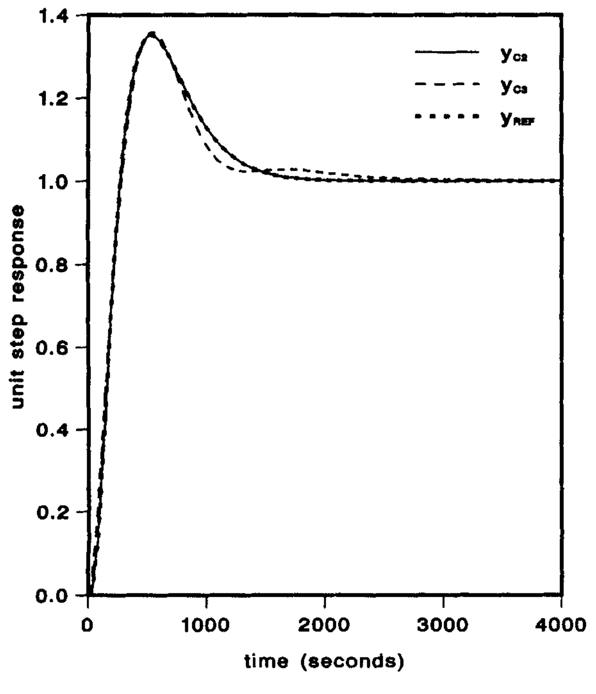

Figure 6. Step responses for example 2: $G_{C2}(s)$ and $G_{C3}(s)$ design.

Conclusion

A novel methodology for process controller design is presented. Closed-loop performance is predetermined in the form of a reference model for the overall system input-output behavior. The key contribution of this paper is the mathematical formulation of the restrictions which must be placed on reference models for integrating and runaway processes. These constraints are the necessary and sufficient conditions for total stability. Equally important are the techniques which supply models for a given process which meet these restrictions. Once the reference model is selected, the controller design is simple and straightforward. The desired behavior is then achieved by placing the process and controller in the unity-feedback configuration of Figure 1. The examples demonstrate the simple, yet flexible, application of the methodology to stable closed-loop process control.

As discussed above, an important topic currently under investigation is the development of guidelines for optimizing reference model response speed without sacrificing robustness or overall system performance. Preliminary results have elucidated some fundamental quantitative guidelines which can be applied. Another important extension of the techniques presented herein is in the area of multivariable systems. Matrix methods have been developed in order to select reference models for total stability and determine the corresponding controllers. Both of these areas are explored in detail in (Quinn, 1988).

The technique of frequency domain model matching provides a direct route to achieve design goals with output feedback and practical controllers (Sanathanan and Quinn, 1987). The reference model selection procedure for total stability presented in this paper extends the technique to processes which are open-loop unstable. The entire class of stable unity-feedback systems can thus be parameterized for a given integrating or runaway process. These results constitute a powerful, practical design approach for process control.

Notation

A_D = reference model *TF* denominator polynomial
A_N = reference model *TF* numerator polynomial
a = coefficient of A_D
b = coefficient of N_+
D = process model denominator polynomial
D_- = factor of D containing all open *LHP* roots
D_+ = factor of D containing all closed *RHP* roots
d = coefficient of D_+
deg = degree of a polynomial
E = free polynomial in Diophantine equation general solution
e = base of natural logarithm
F = one of Diophantine equation polynomial solution pair: $\{F, G\}$
F_0 = one of minimum-degree Diophantine polynomial solution pair: $\{F_0, G_0\}$
f = coefficient of F or F_0
G = one of Diophantine equation polynomial solution pair: $\{F, G\}$
G_0 = one of minimum-degree Diophantine polynomial solution pair: $\{F_0, G_0\}$
g = coefficient of G or G_0
G_{APPROX} = approximate process model *TF*
G_C = controller *TF*
G_{CL} = closed-loop system *TF*
G_F = filtered reference model *TF*
G_P = process model *TF*
G_R = reference model *TF*
i = deg $[A_D]$
j = deg $[N_-]$
$j = \sqrt{-1}$
K = ideal controller *TF*
k = deg $[D_-]$
L = loop return ratio
LHP = left half s plane
l = coefficient index
m = deg $[N_+]$
N = process model numerator polynomial
N_- = factor of N containing all open LHP roots
N_+ = factor of N containing all closed RHP roots
n = deg $[N_-]$
P = numerator polynomial of L
Q = denominator polynomial of L
RHP = right half s plane
r = set point input
s = Laplace transform variable
T = process time constant
TF = transfer function
u = process input signal
y = process output signal

Greek letters

α = numerator coefficient of G_{CL}
β = denominator coefficient of G_{CL}
γ = nonzero denominator coefficient of L
δ = power series coefficient in Eqs. 8–11
ι = coefficient index
ν = loop type
Σ = summation operator
τ = dead time
ω = angular frequency

Miscellaneous symbols

$\forall$ = for every
$\rightarrow$ = therefore

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